Disturbance Rejection with a Highly Oscillating Second-order-like Process, Part VI: PPI Controller

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Abstract—A PPI controller is investigated for disturbance rejection associated with a highly oscillating second-order-like process. The controller is tuned using the MATLAB optimization toolbox for different error-based objective functions. The best objective function suitable for this type of controllers for the highly oscillating second order process is assigned. The unit step disturbance input time response of the control system has zero steady-state error and low response levels. The effect of the proportional gain of the PPI controller on the system dynamics is investigated. The PPI controller when used for disturbance rejection associated with the highly oscillating second-order process can compete well with PD-PI and PI-PD controllers used for the same purpose.

Keywords—Disturbance rejection, PPI controller, second-order-like process, controller tuning.

I. INTRODUCTION

This is the sixth paper in a series of research papers aiming to investigate specific controllers and compensators for disturbance rejection associated with second-order-like processes having high oscillation nature. The resulting control system has to be stable and capable of rejecting the disturbance input with good performance measures.

Rada and Lo (1994) proposed using a predictive proportional integral controller with improved performance. They realized the PPI controller by continuous time implementation [1]. Ren, Zhang and Shao (2003) demonstrated the performance of PPI controller and showed that the PPI controller is suitable for processes having high oscillation nature. The PPI controller when used for disturbance rejection associated with second-order-like processes having high oscillation nature. The PPI controller tuning. The effect of the proportional gain of the PPI controller on the system dynamics is investigated. The PPI controller when used for disturbance rejection associated with the highly oscillating second-order process can compete well with PD-PI and PI-PD controllers.
compared the control system performance with control systems using I-PD, PD-PI, PI-PD, PID + first-order lag and PID controllers [11].

II. PROCESS

The process is a second-order like one without time delay having the transfer function, $G_p(s)$:

$$G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$  \hspace{1cm} (1)

Where:

$\omega_n$ = process natural frequency in rad/s = 10 rad/s
$\zeta$ = process damping ratio = 0.05

This process has a maximum overshoot of 85.4% indicating the high oscillation nature of the process.

III. THE PPI CONTROLLER

The block diagram of a linear feedback control system for set-point tracking exhibiting a PPI controller is shown in Fig.1 [12].

![Fig.1 PPI controller in a control system for set-point tracking](image)

The PI controller part in Fig.1 is a standard PI controller having the transfer function, $G_{PI}(s)$:

$$G_{PI}(s) = K_{pc} + \frac{K_i}{s}$$  \hspace{1cm} (2)

According to Airrikka, the PPI controller of the structure shown in Fig.1 has an overall transfer function between its output $U(s)$ and input $E(s)$, $G_{PPI}(s)$ given by [12]:

$$G_{PPI}(s) = \frac{s(K_{pc} + K_i/s)}{s + K_{pred}[1 - \exp(-Ls)]}$$  \hspace{1cm} (3)

Where:

$K_{pc}$ and $K_i$ are the proportional and integral gain coefficients of the PI controller.
$K_{pred}$ is the predictive gain coefficient of the feedback element shown in Fig.1.
$L$ is the time delay of the PPI controller in the feedback element shown in Fig.1.

To facilitate the dynamic analysis of the control system, the first-order Taylor series is used to replace the exponential term in Eq.3 by a first-order polynomial. That is [13]:

$$\exp(-Ls) = -Ls + 1$$  \hspace{1cm} (4)

Combining Eqs.3 and 4 gives:

$$G_{PPI}(s) = \frac{(K_{pc} + K_i/s)}{1 + K_{pred}L}$$  \hspace{1cm} (5)

The term 1 + $K_{pred}L$ in Eq.5 is a constant parameter independent of the Laplace operator $s$. Therefore, it can be replaced with one parameter $K'$. That is:

$$G_{PPI}(s) = \frac{(K_{pc} + K_i/s)}{K'}$$  \hspace{1cm} (6)
IV. CLOSED-LOOP TRANSFER FUNCTION

The closed loop transfer function of the closed loop control system depends on the input and output variables of the control system. With disturbance input included in the analyses, the control system has two input variables: reference input $R(s)$ and disturbance input $D(s)$ as shown in Fig.2.

![System block diagram with two input variables.](image)

For sake of studying the dynamics of the control system for disturbance rejection, only the disturbance variable $D(s)$ has to be considered, and the reference input will be omitted from Fig.2. In such a case, the controller block will come in the feedback path of the single loop of the control system and the controller output signal will enter the error detector (summing point) with a negative sign. The transfer function of the control system in this case, $C(s)/D(s)$ considering Eqs. 1 and 6 will be:

$$C(s)/D(s) = \frac{b_0 s}{s^3+a_0 s^2+a_1 s+a_2}$$

Where:

- $b_0 = \omega_n^2$
- $a_0 = 2\zeta \omega_n$
- $a_1 = \omega_n^2 (1 + \frac{K_{pc}}{K'})$
- $a_3 = \omega_n^2 K_i / K'$

V. CONTROLLER TUNING

Tuning of the PPI controller for disturbance rejection of the highly oscillating second-order process allows adjusting the controller three parameters $K_{pc}$, $K_i$ and $K'$ for optimal disturbance rejection. The desired steady-state response for disturbance rejection is zero. This allows us to define an error function $e(t)$ as the time response to the unit disturbance input. That is:

$$e(t) = c(t)$$

The controller tuning is performed using the error function of Eq.8 which is incorporated in an objective function to be minimized using the MATLAB optimization toolbox [14]. The objective functions used are ([15]-[17]):

- ITAE: $\int t |e(t)| \, dt$
- ISE: $\int [e(t)]^2 \, dt$
- IAE: $\int |e(t)| \, dt$
- ITSE: $\int t |e(t)|^2 \, dt$
- ISTSE: $\int t^2 |e(t)|^2 \, dt$

The tuning results for a control system incorporating the PPI controller and the highly oscillating second-order-like process for disturbance rejection are given in Table 1.

<table>
<thead>
<tr>
<th>$K_{pc}$</th>
<th>ITAE</th>
<th>ISE</th>
<th>IAE</th>
<th>ITSE</th>
<th>ISTSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>$K_i$</td>
<td>1.1019</td>
<td>1.1019</td>
<td>0.10</td>
<td>0.10</td>
<td>1.1003</td>
</tr>
<tr>
<td>$K'$</td>
<td>3.9745</td>
<td>3.9749</td>
<td>3.9917</td>
<td>4</td>
<td>3.9751</td>
</tr>
<tr>
<td>$c_{max}$</td>
<td>0.0077</td>
<td>0.0077</td>
<td>0.0077</td>
<td>0.0077</td>
<td>0.0077</td>
</tr>
<tr>
<td>$T_{cmax} (s)$</td>
<td>12.1157</td>
<td>12.1157</td>
<td>133.44</td>
<td>133.28</td>
<td>12.1338</td>
</tr>
</tbody>
</table>
It is clear from the tuning results in Table 1 that the ITAE, ISE and ISTSE objective functions generate almost the same optimal time response while the IAE and ITSE generate another optimal sluggish time response.

VI. CONTROL SYSTEM TIME RESPONSE

The time response of the control system for a unit step disturbance input for the five objective functions of Eqs.9 to 13 is shown in Fig.3.

Fig.3 Unit step disturbance input time response using a PPI controller.

Fig.3 indicates that the step time response is smooth, has a small level and decays to zero in a very long time. However, it has a zero settling time as it is less than an 0.05 value.

VII. EFFECT OF CONTROLLER PROPORTIONAL GAIN $K_{pc}$

Different levels of the proportional gain $K_{pc}$ are tried keeping the other optimal values in Table I. Since the optimization problem of the tuning process is nonlinear in the controller parameters, local minima are expected. Different levels of $K_{pc}$ are tried keeping the other PPI controller parameters at the levels in Table I. The simulation results investigating this effect are shown as an effect of the time response of the control system due to the unit step disturbance input shown in Fig.4.

It is clear from Fig.4 that $K_{pc}$ has a great effect on the disturbance rejection process. It is possible to decrease the maximum time response value from 0.0077 to 0.00096 by increasing the gain $K_{pc}$ from 500 to 4000.
The effect of the PPI proportional gain is further investigated through the maximum time response, $c_{\text{max}}$ and its time, $T_{c_{\text{max}}}$. This effect is illustrated graphically in Fig.5 for proportional gain in the range: $500 \leq K_{pc} \leq 4000$.

It is clear how the maximum time response decreases as the proportional gain increases while the time of the maximum response increases as the proportional gain increases.
VIII. COMPARISON WITH OTHER CONTROLLERS
To investigate the effectiveness of using a PPI controller for disturbance rejection associated with a second-order-like highly oscillating process it is compared with that of using PD-PI [18], PI-PD [19], IPD [20] and 2DOF [21] controllers for the same process. The unit step disturbance input time response of the control system using the compared five controllers is shown in Fig.6.

Fig.6 Comparison of the unit step disturbance input time response.

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>c&lt;sub&gt;max&lt;/sub&gt;</th>
<th>T&lt;sub&gt;cmax&lt;/sub&gt; (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD-PI controller</td>
<td>0.0950</td>
<td>0.1200</td>
</tr>
<tr>
<td>PI-PD controller</td>
<td>0.0013</td>
<td>0.0100</td>
</tr>
<tr>
<td>IPD controller</td>
<td>1.927*10^-4</td>
<td>0.0036</td>
</tr>
<tr>
<td>2DOF controller</td>
<td>5.113*10^-4</td>
<td>0.0079</td>
</tr>
<tr>
<td>PPI controller</td>
<td>9.60*10^-4</td>
<td>37.580</td>
</tr>
</tbody>
</table>

IX. CONCLUSIONS
- A PPI controller was investigated for disturbance rejection associated with a highly oscillating second-order-like process.
- The controller was tuned using MATLAB optimization toolbox and five different objective functions.
- The ITAE, ISE and ISTSE objective functions gave the same effect on the time response of the control system.
- The effect of the proportional controller of the PPI controller on the disturbance time response was investigated where it had a remarkable effect on the time response due to disturbance input.
- The PPI controller could go down with the maximum time response value to as low as 9.6*10^-4.
- The time response to a step disturbance input was sluggish within its very small value.
- Comparing with the research work using PD-PI, PI-PD, IPD and 2DOF controllers, the PPI controller could compete with the PD-PI and PI-PD controllers regarding the maximum time response value.
REFERENCES

BIOGRAPHY

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- Emeritus Professor of System Dynamics and Automatic Control.
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